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ISMM 07

Rio de Janeiro
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Random spread and Forest Fires

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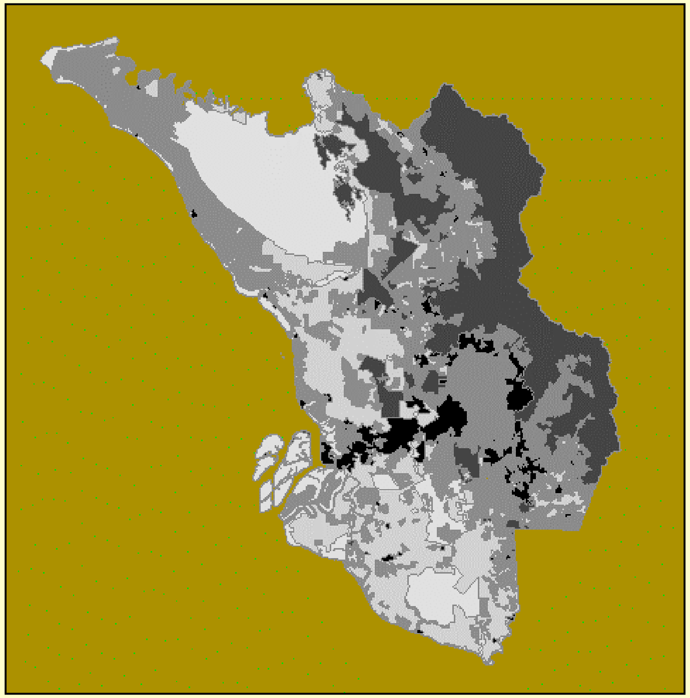
Classical maps

- Usual maps symbolize objects that ***do exist*** in the physical word,
- e.g. the Malaysian peninsula is ***prior*** to any geographer, and independent of him.

Risks maps

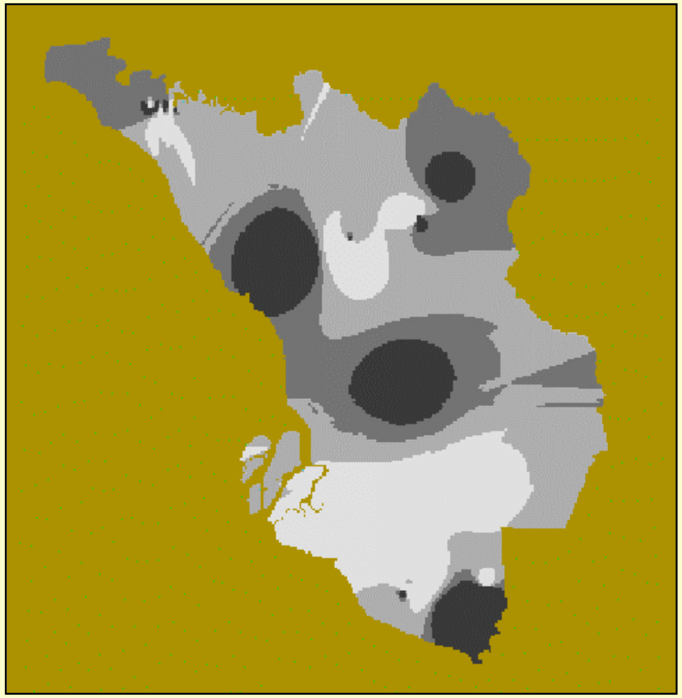
Consider now the two maps of forest fires parameters

Fuel amount : θ



Relative Scale :
the lighter, the more fuel

Fire spread : ρ



Scale : 0 meter/mn (black)
to 3 meters/mn (light)

Selangor State, Malaysia

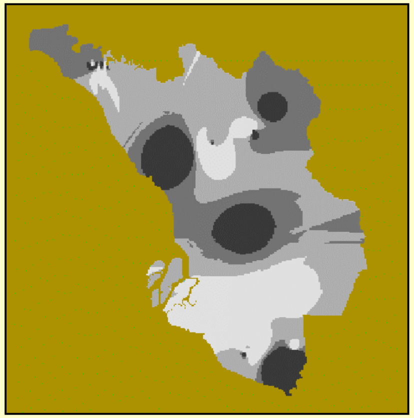
A missing link

- When we draw the map of a risk, e.g. a spread fire, we describe a ***scientific assumption***.
- There is ***no actual object*** in the physical word that the map symbolizes: it represents potentialities only.
- If we want to go from potentialities to the actual events, an ***additional element*** turns out to be necessary.

…..But how to handle it ?

The missing link

Fire spread : ρ



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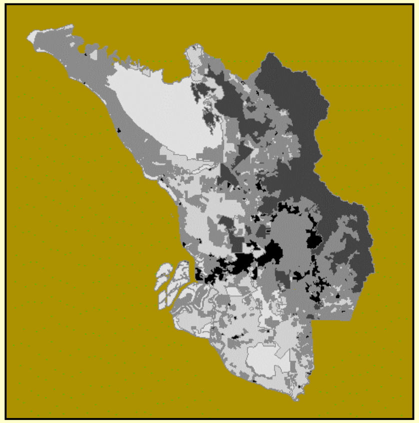
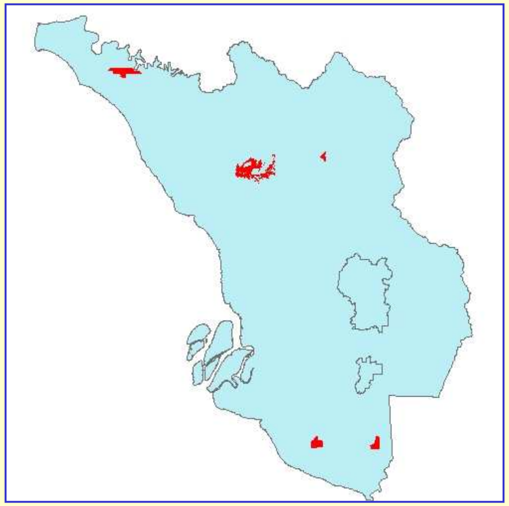


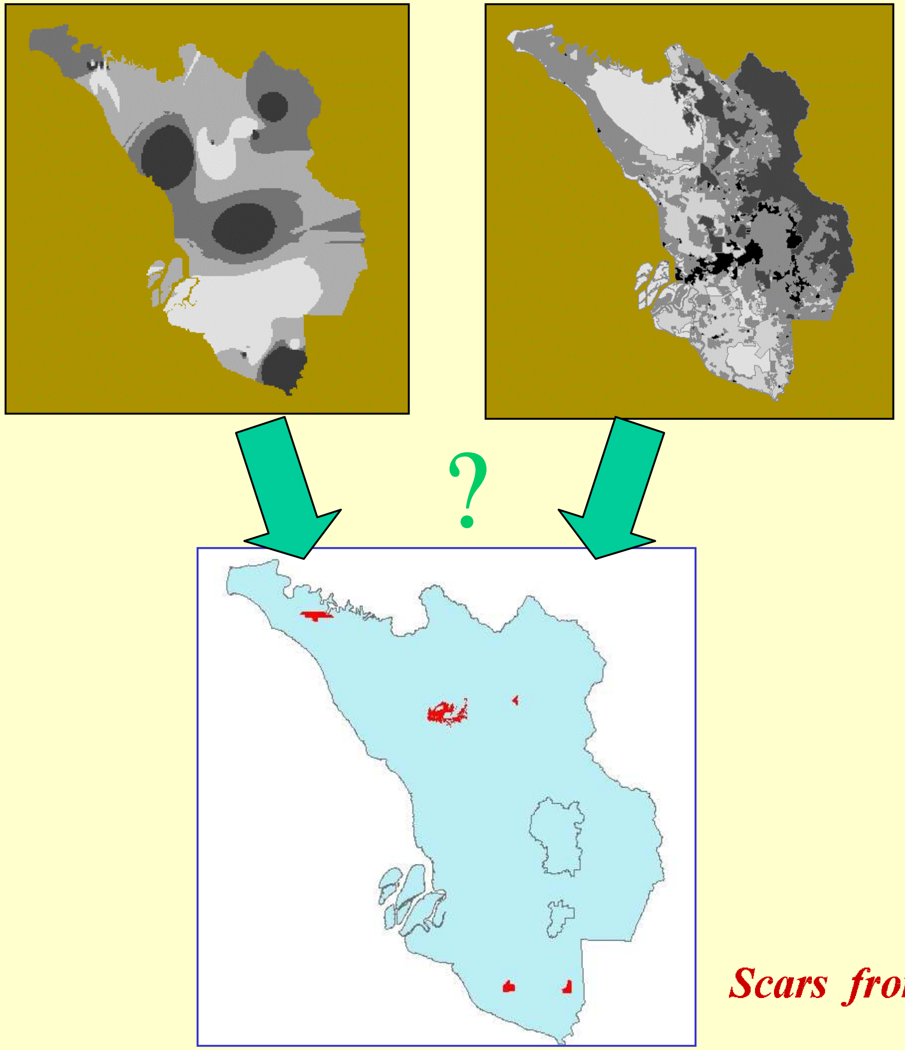
- How to go from these two maps to the burnt regions?
- Can we derive from them the duration of a fire ?
- and the size distribution of the burnt regions ?
- **Without a model, surely not !**



Scars from 2000 to 2004

The missing link

Fire spread : ρ   *Fuel amount :* θ



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A Pde ?

- Classically, each Pde summarizes a conflict of elementary variations,
- but here, we are facing a space-time process whose all parameters act in the sense of the space invasion.

How to introduce an element that balance the invasion?

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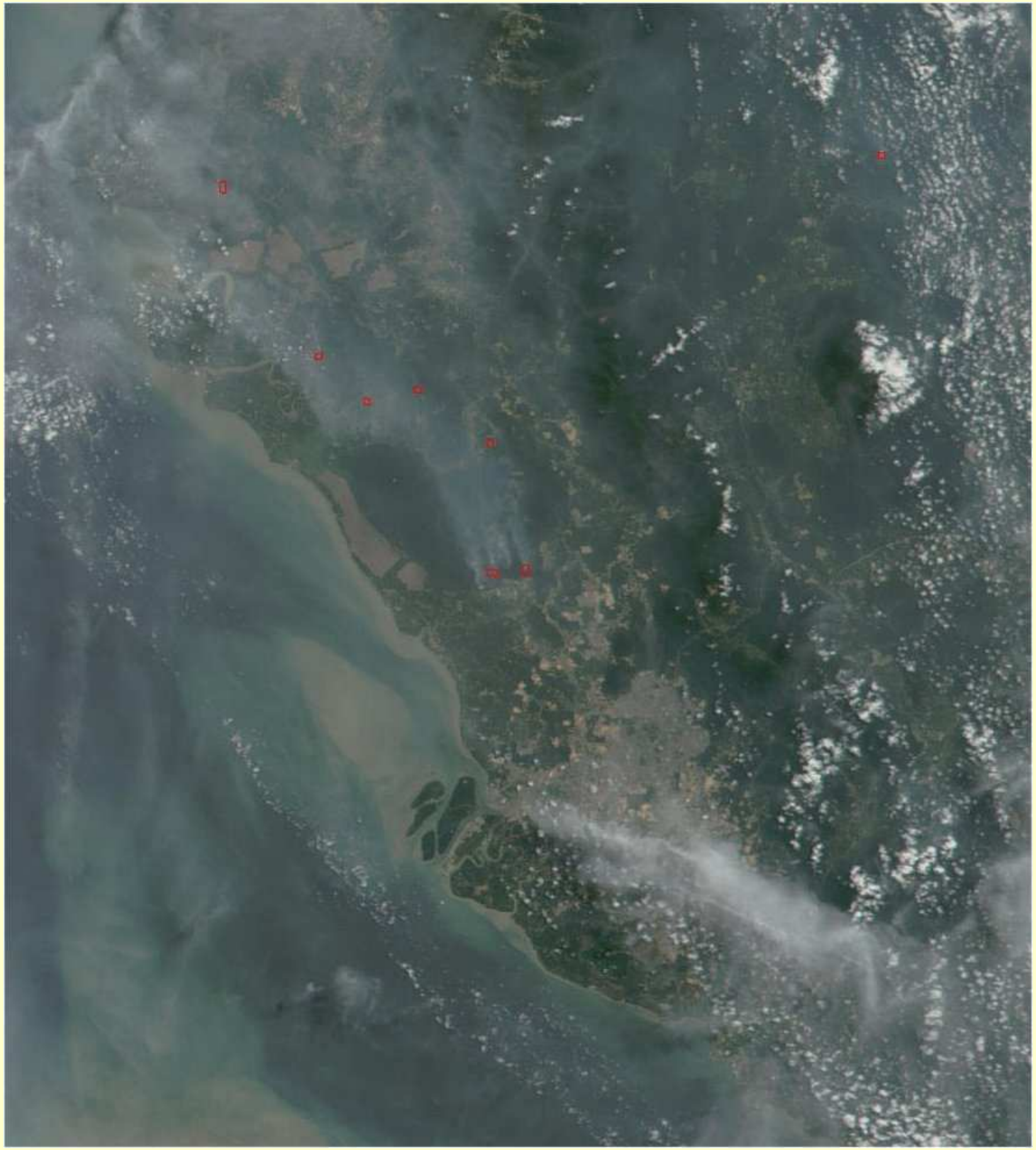
How to introduce an element that balance the invasion?

⇨ the ***hot spots*** provide a third piece of information.

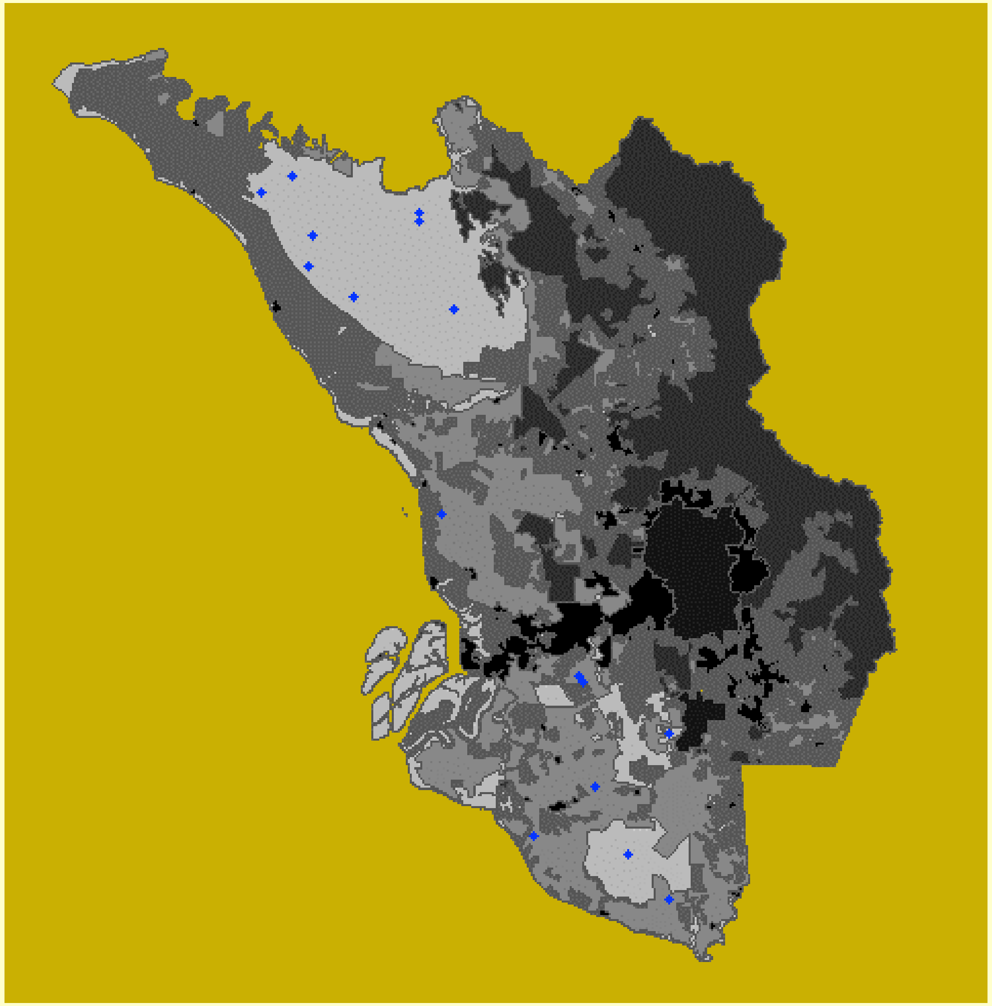


Hot Spots

In red, the hot spots detected in Selangor, on August 12, 2005

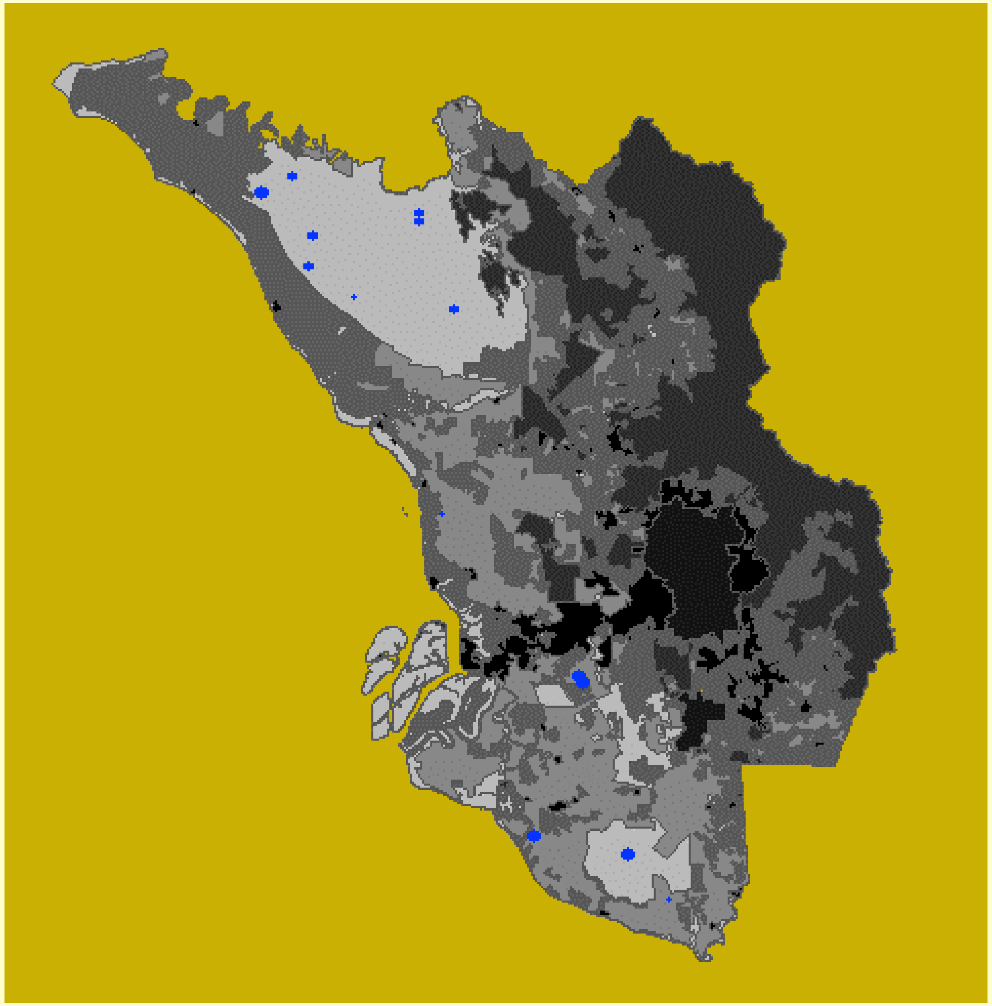
The spread map based Pde

Initial hot spots



The spread map based Pde

The speed of expansion is given by the spread rate map



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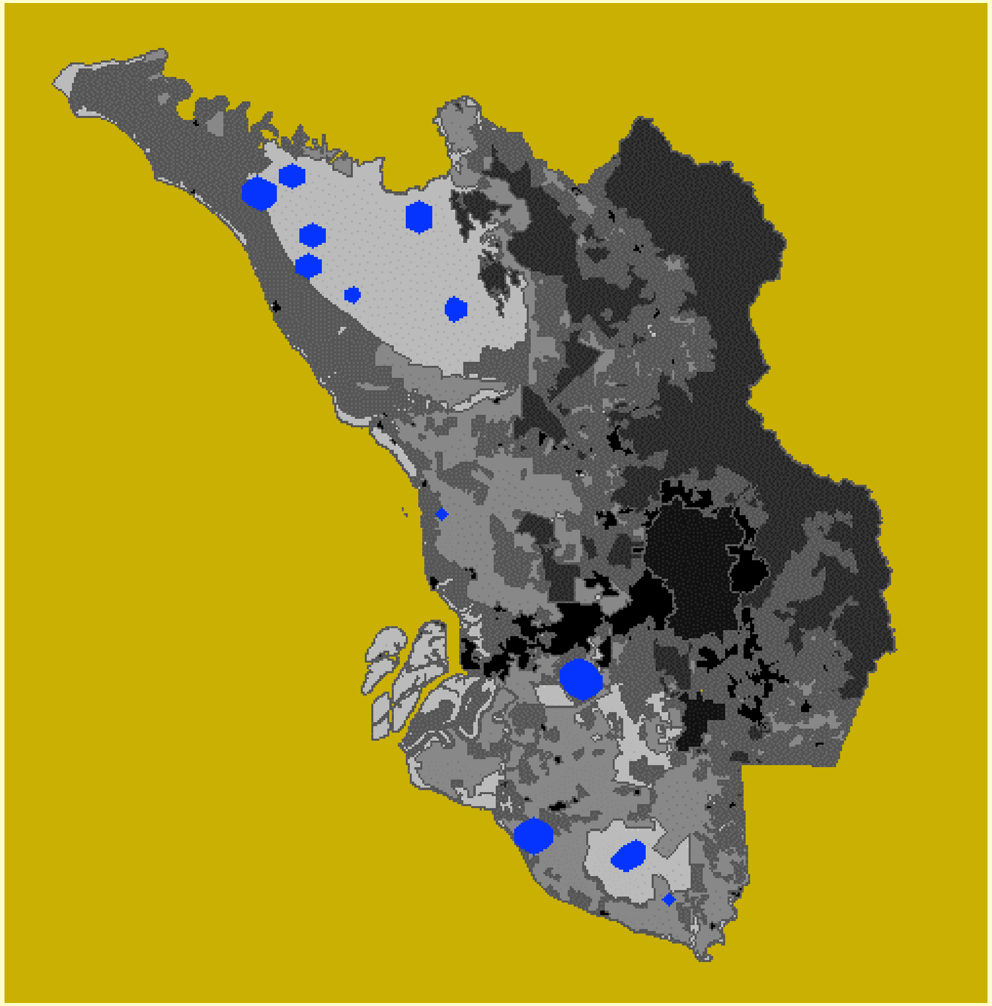

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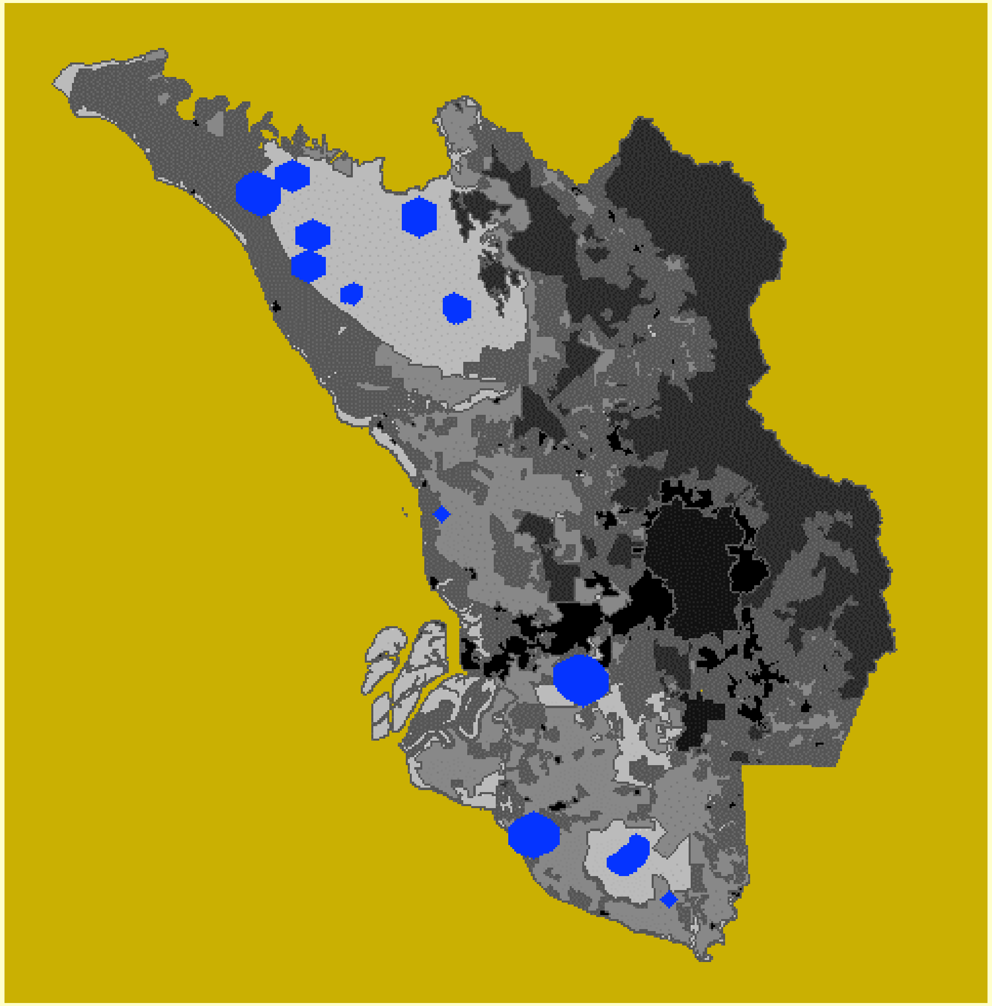

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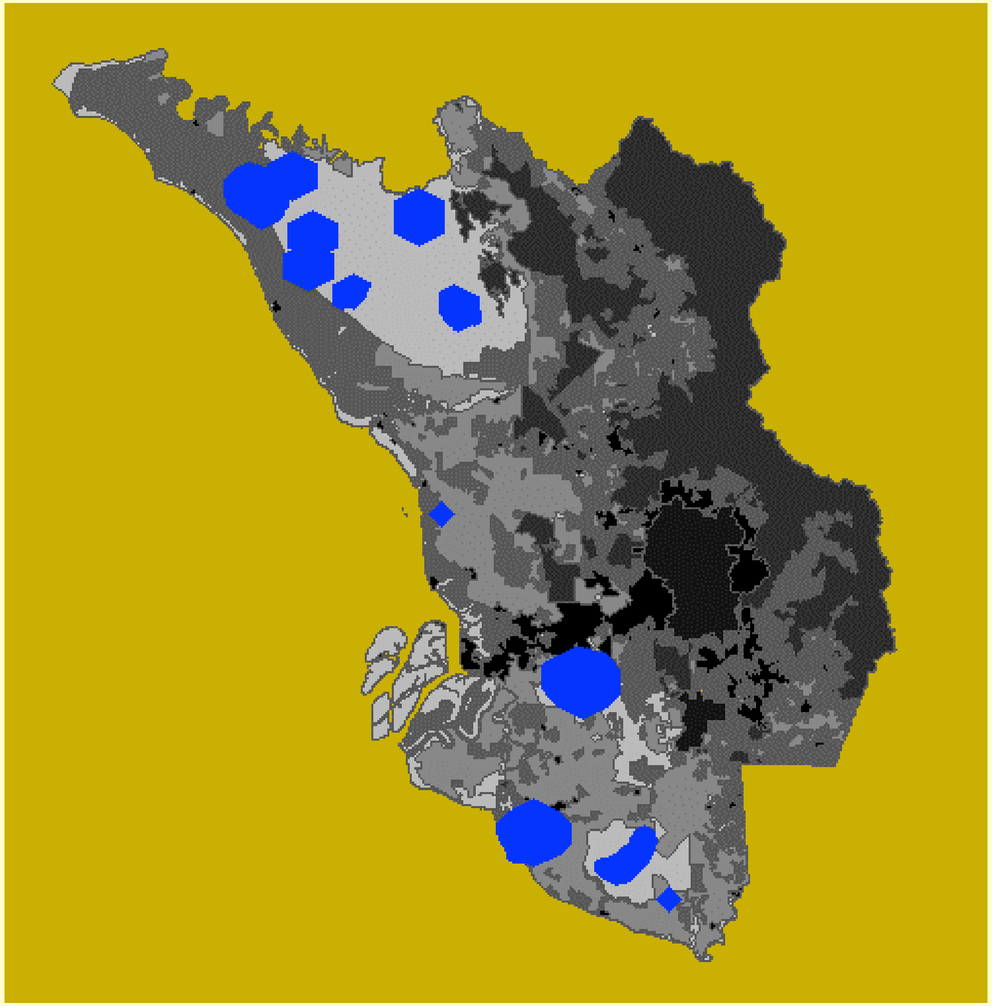
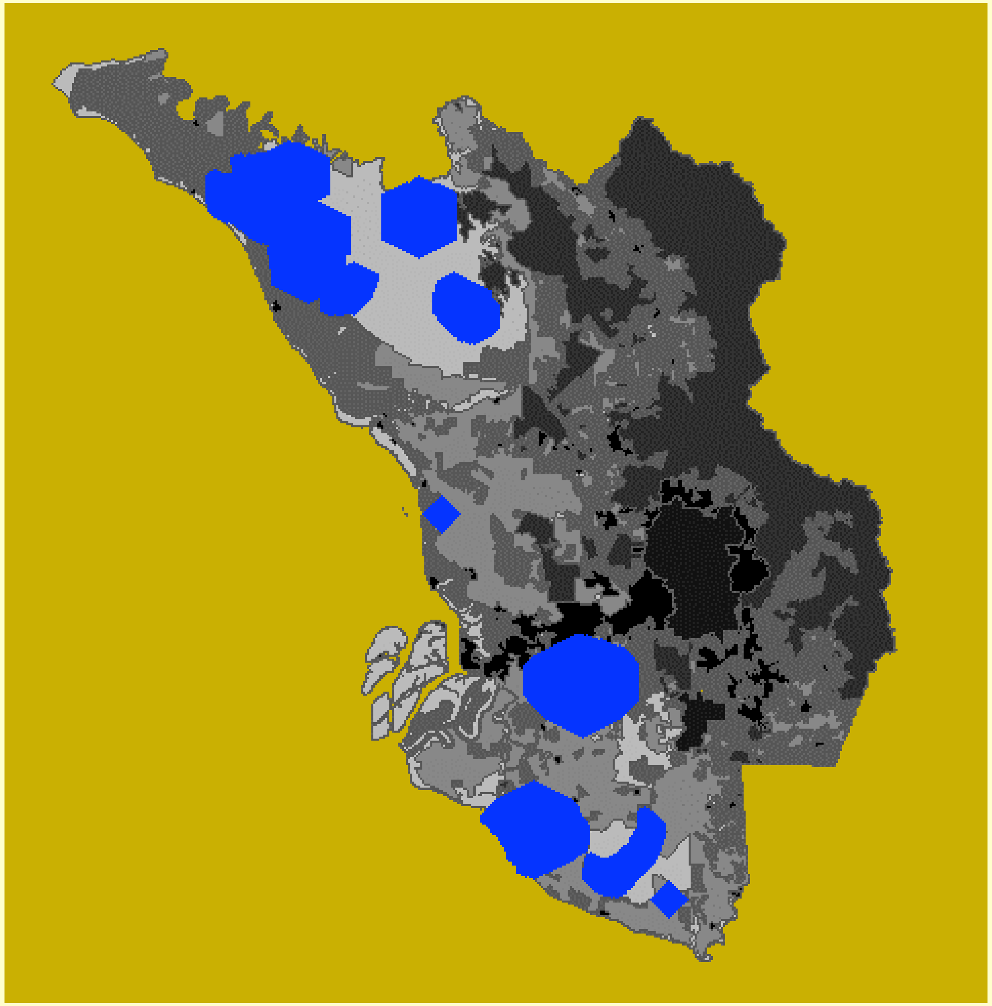


The spread map based Pde

When the seats
expand according to
the spread map,
then they progressively
invade the whole space



A theoretical tool

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  - ⇒ daily evolution of the fire;
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- It results in ***predictions*** for the
  - ⇒ daily evolution of the fire;
  - ⇒ the possible natural extinction,
  - ⇒ location of the scar regions.
- Finally, it also leads to ***simulations*** of the fire propagation.

Random Closed Sets (or RACS)

Let $\mathbb{R}^d$ be the Euclidean space of dimension d,

$\mathcal{F} = \mathcal{F}(\mathbb{R}^d)$ denotes the family of all closed sets of $\mathbb{R}^d$,

$\mathcal{K} = \mathcal{K}(\mathbb{R}^d)$ the family of all compact sets.

- **σ-algebra** : Given an element $K \in \mathcal{K}$, consider the class $\mathcal{F}(K)$ of all closed sets that miss the compact set K. As K spans the family $\mathcal{K}$, the classes $\{\mathcal{F}(K), K \in \mathcal{K}\}$ generate a σ-algebra.
- **RACS** : Moreover, as $\mathcal{F}$ is a compact space, one can weight σ by probabilities P. Then each triplet (F,σ,P) ***defines a RACS***.

This abstract definition of a RACS goes back to G.Matheron and D.G.Kendall. However, these authors made their approach more tractable by proving the following result.

The Matheron-Kendall theorem

- ***Characteristic Theorem*** : Every RACS X is characterized by the datum of the probabilities

$$\mathbf{Q(K) = Pr\{ K \subseteq X^c \} \qquad K \in \mathcal{K} .}$$

Conversely, a family $\{Q(K), K \subseteq \mathcal{K}\}$ defines a unique RACS if and only if 1- Q(K) is a **Choquet capacity** such that

$$0 \leq Q \leq 1 \quad \text{and} \quad Q(\emptyset) = 1.$$

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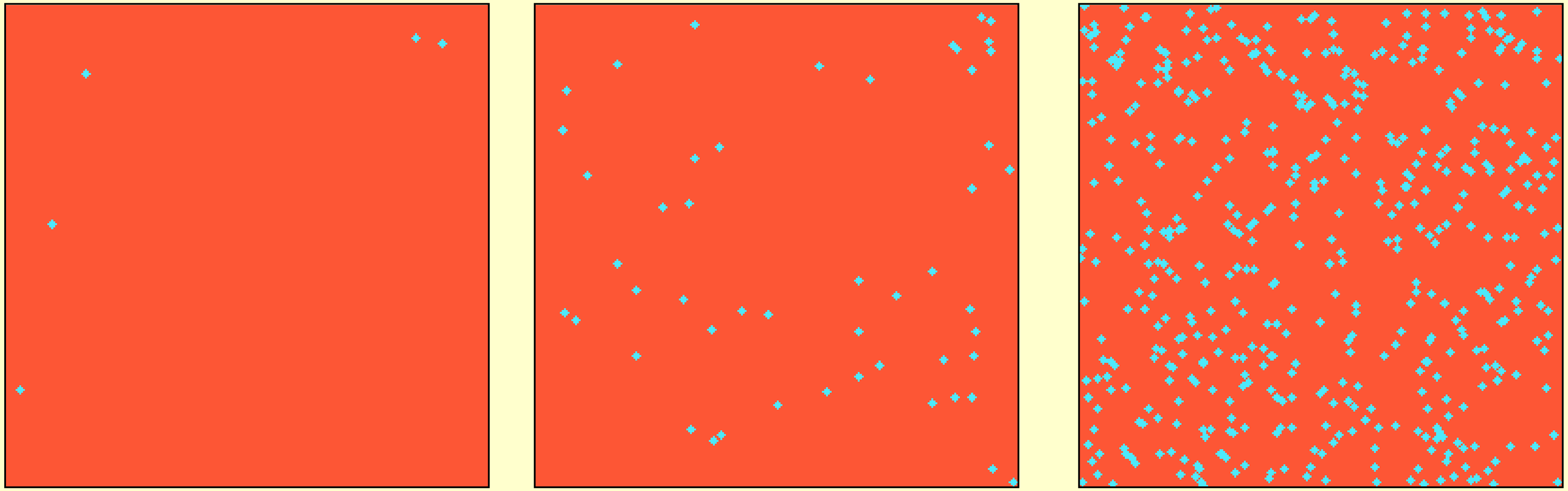
- ***Choquet' capacity*** : numerical function Q on $\mathcal{K}$ such that

1- $S_1(K; K_1) = Q(K) - Q(K \cup K_1)$
$S_n(K; K_1..K_n) = S_{n-1}(K; K_1..K_{n-1}) - S_n(K \cup K_n; K_1..K_{n-1})$

2- $K_n \downarrow K$ implies $Q(K_n) \uparrow Q(K)$

Poisson points

- Here are usual simulations of Poisson points (slightly dilated by a rhomb for the display)



They are «***usual***» in that the intensity $\theta(x)$ is ***constant***

Poisson points

A basic random set is that of the Poisson points, defined as follows

- 1/ If B and B' are ***disjoints***, then the numbers of points in B and B' are ***independent*** variables;

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- 1/ If B and B' are ***disjoints***, then the numbers of points in B and B' are ***independent*** variables;

- 2/ If B = dx is a small set, then the probability of

1 point in dx is **$\theta(dx)$**

0 point in dx is **$1- \theta(dx)$**

The functional Q(K) of Poisson points θ is

$$Q(K) = \exp\{-\int_K \theta(dx) \} = \exp\{ - \theta(K) \}$$

Regionalized Poisson points

In some cases the intensity of the Poisson points can also ***vary*** through the space …

We still have the probability

θ(x) dx

of one point in dx,

but ***θ*** is now an underlying function of the space

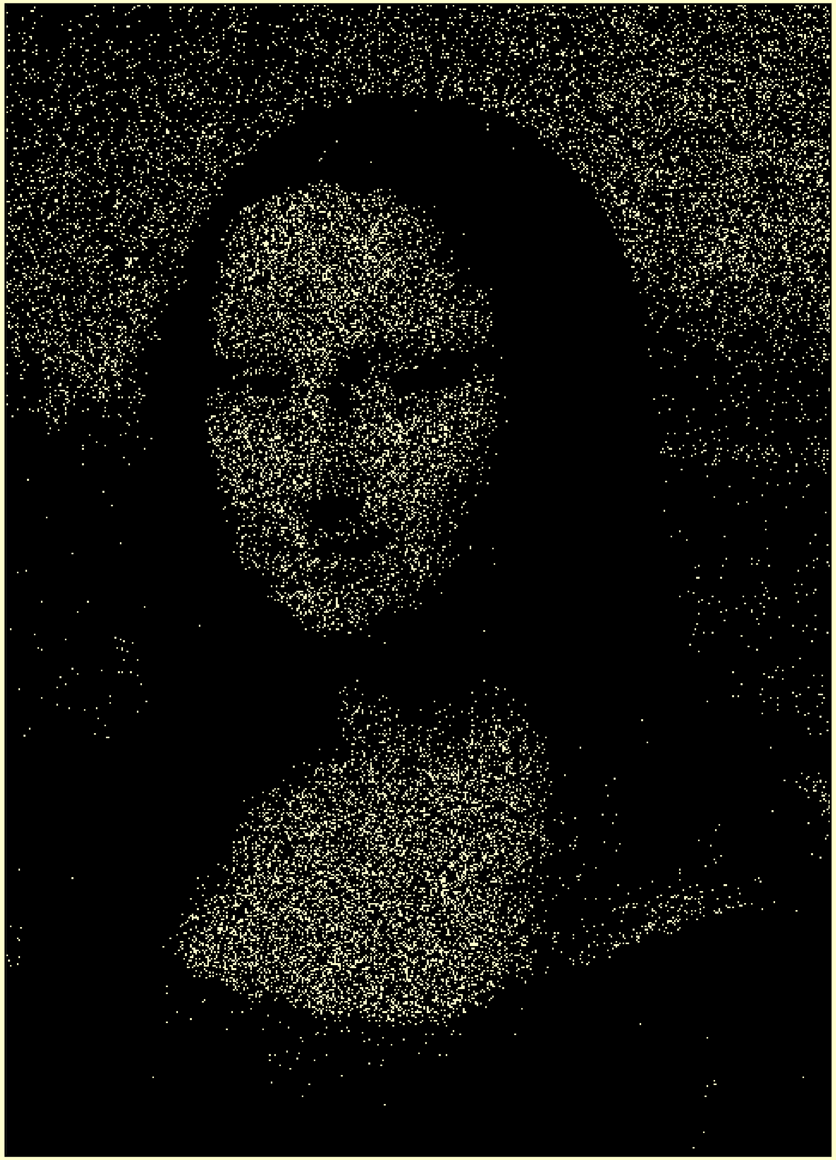
Regionalized Poisson points

As θ is multiplied by a constant factor,

the number of points increases

Regionalized Poisson points

And more again…



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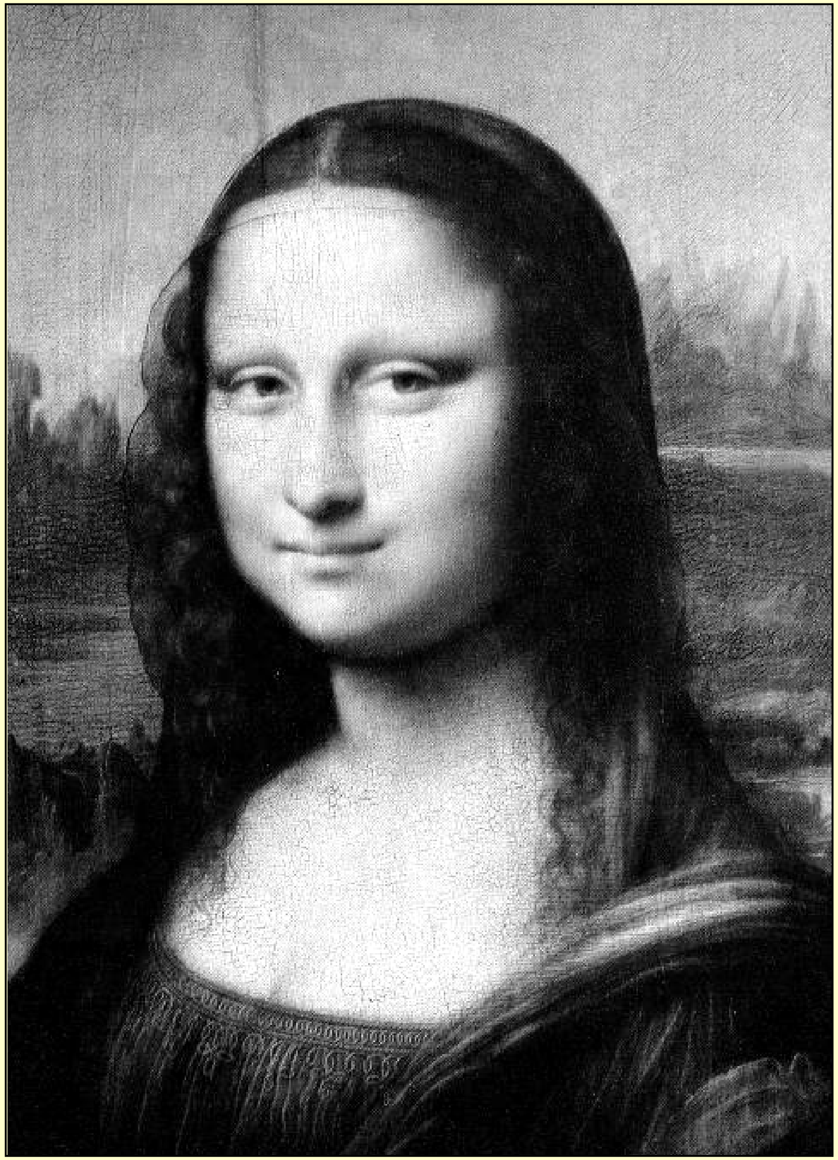
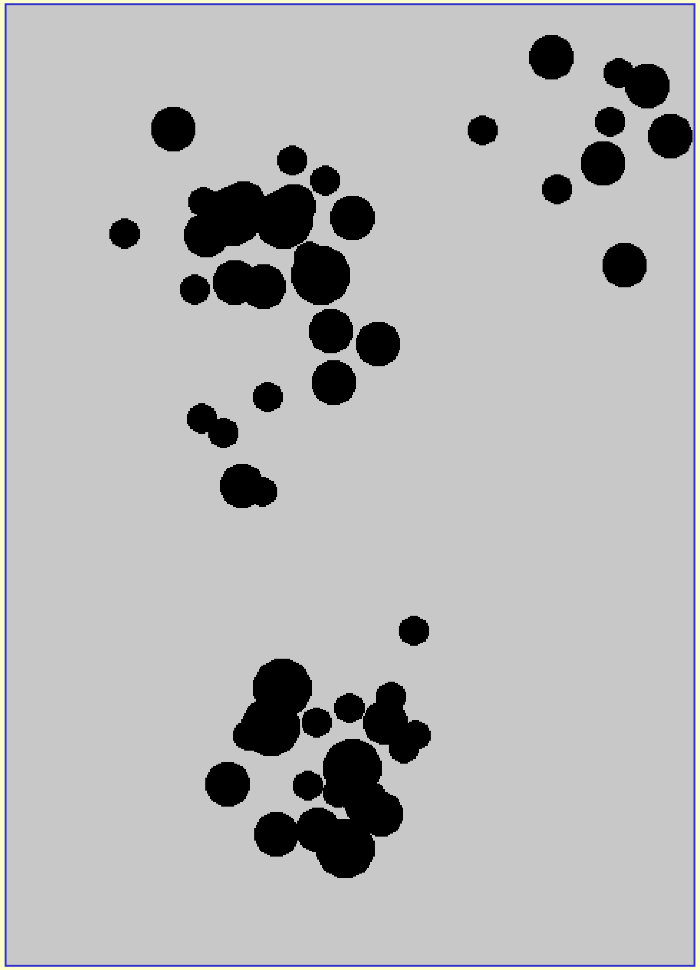


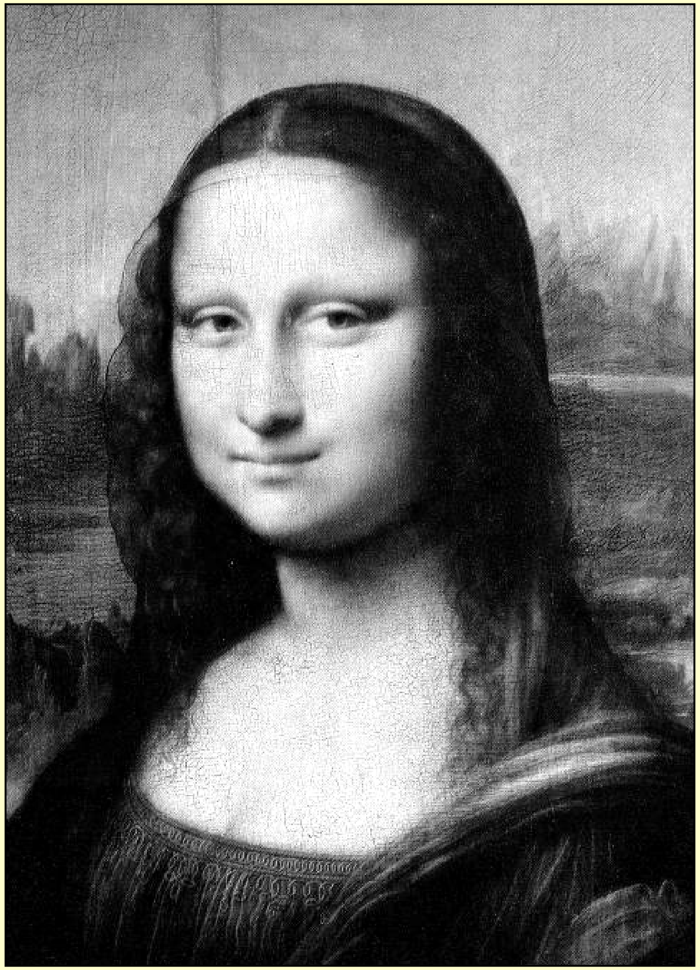
Boolean random set

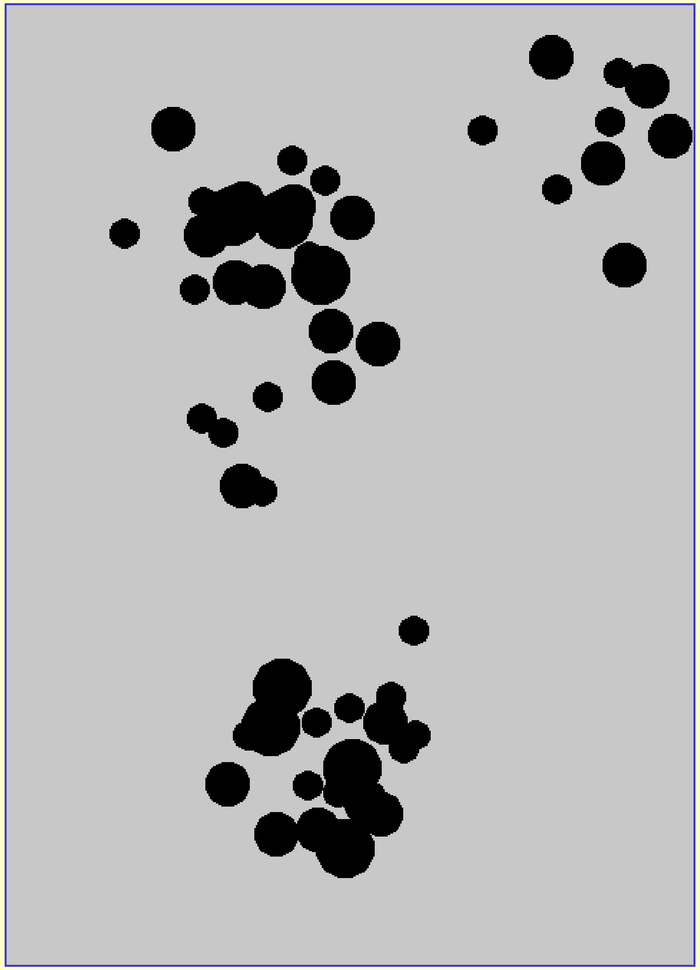


variable intensity
$\theta(x)$

variable primary grain
$\delta(x)$

Boolean random set





variable intensity $\theta(x)$

variable primary grain $\delta(x)$

Two parameters

Just as a Boolean random set , a random spread depends on

- The ***the intensity*** θ ***, non negative numerical function***
- The ***dilation*** δ***, a set function*** $\mathbb{R}^d \rightarrow \mathcal{P}(\mathbb{R}^d)$

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- The ***the intensity θ , non negative numerical function***
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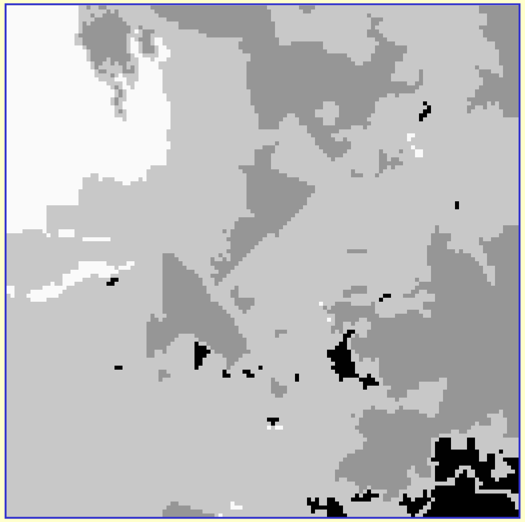
Here we take for set δ(x) the disc of radius ρ at point x



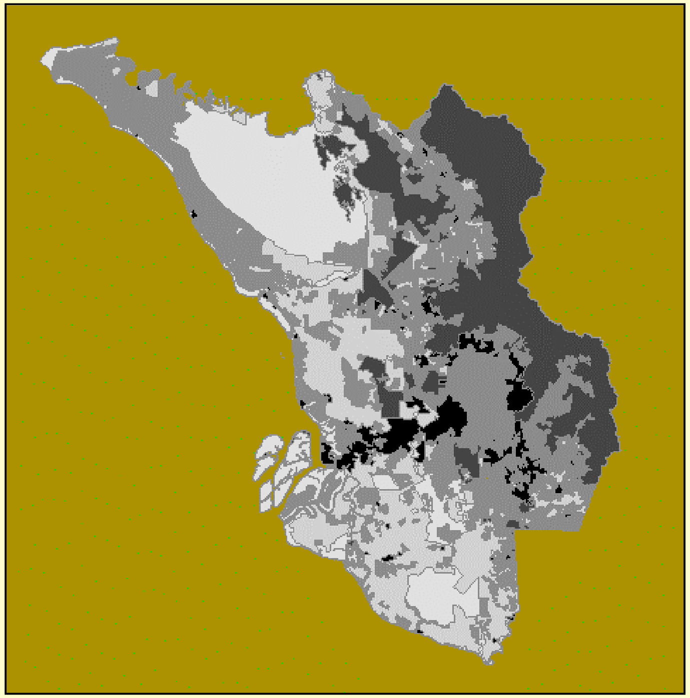
Point intensity θ

Dilation radius ρ

Parameters maps

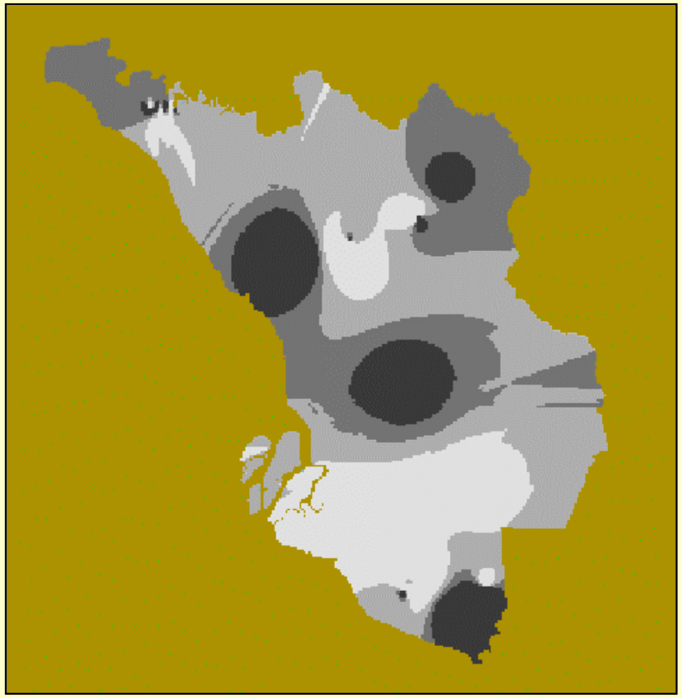
The two previous maps are details of the following risk maps

Fuel amount : θ



Relative Scale :
the lighter, the more fuel

Fire spread : ρ



Scale : 0 meter/mn (black)
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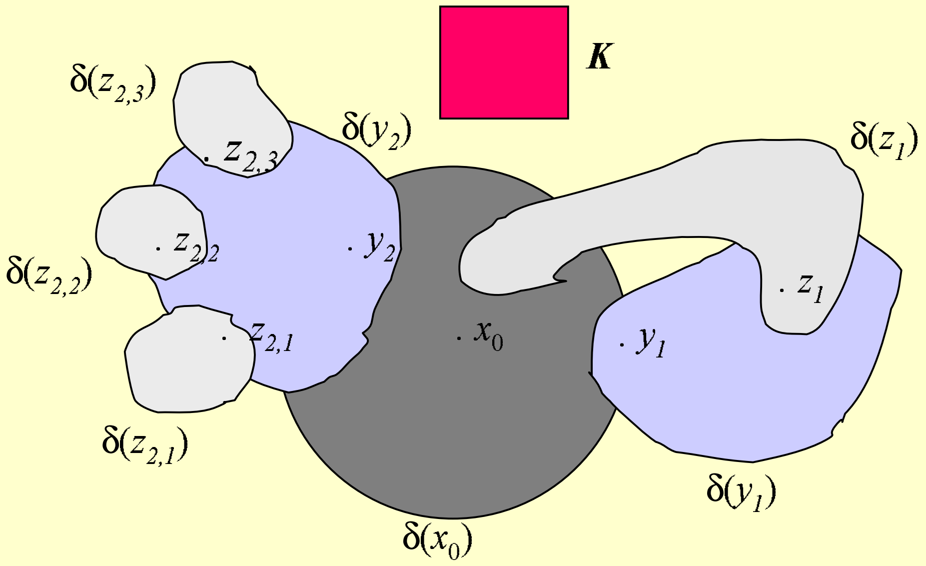
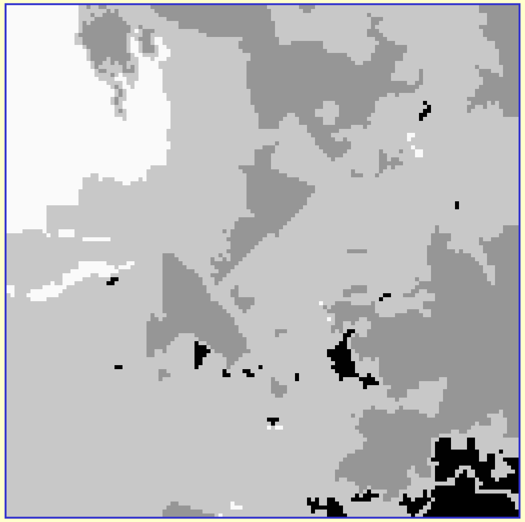
(Selangor State, Malaysia)

Random Spread

The idea of a random spread is the following



Iterated spread



Point intensity

Dilation radius

Fire $X_n = \delta(I_{n-1}) = \delta \circ [\beta]^{n-1} (I_0)$

Seat $I_n = \beta^n(I_0) = \cup \{ \delta(x_i) \cap J_i , x_i \in I_1 \}$

Examples of iterated Spread

Functional of the Boolean set

The Boolean Random set $X(\theta, \delta)$ is characterized by the probabilities $Q(K)$ that K misses the RACS , for all compact sets $K \subset \mathbb{R}^d$ (Choquet characteristic). We have that

$$\mathbf{Q(K)} = \exp \{ - \int_{\zeta(K)} \theta(dx) \} = \exp \{ - \theta[\zeta(K)] \}$$

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where ζ is the ***reciprocal*** of δ, i.e.

$$x \cap \zeta(K) \neq \emptyset \quad \Leftrightarrow \quad \delta(x) \cap K \neq \emptyset$$

Functional of iterated spread

Let us calculate the functionals $Q_1 \dots Q_n$ of spreads $X_1 \dots X_n$.

- The first step is just Boolean, so that the Choquet characteristic

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- Now, to say that K misses the n^{th} fire starting from x is equivalent to saying that K misses the $(n-1)^{th}$ fire from y, cond. upon $y \in \delta(x_0)$. This results in the induction relation

$$Q_n(K) = \exp [1 - \int_{\zeta(x)} \theta(dy) Q_{n-1}(K \mid y)]$$

Reciprocal dilation

- ***Reciprocal dilation***: Again we meet the reciprocal dilation ζ of δ i.e. such that

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- ***Translation invariance:*** when $\delta(\mathbf{x})$ is the translate of a symmetrical convex set, then things are simple, as

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$$\delta(\mathbf{x}) = \zeta(\mathbf{x}) \qquad \mathbf{x} \in \mathbb{R}^{\mathbf{d}} .$$

- ***Variable*** δ But in the application to forest fires, δ varies from 1 to 5 from place to place. Which conditions must we demand to δ to get a non trivial expression for

$$\exp\ [\ -\int_{\zeta(\mathbf{K})} \theta(\mathbf{dz})\ \mathbf{g(z)}\ ]\ ?$$

Compact dilation

Here, the convenient class that of the ***compact dilations*** δ.

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Dilation δ is said to be ***compact*** when

1- the structuring function $x \rightarrow \delta(x)$ is u.s.c. from $\mathbb{R}^d$ into $\mathcal{K}$,

2- the union $\cup\{\delta_{-x}(x), x \in \mathbb{R}^d\}$ has a compact closure.

The second axiom implies that when x is far away enough, then $\delta(x)$ surely misses K

Compact dilation

- When δ is compact, then
  - ζ also is compact,
  - δ and ζ are u.s.c. mappings from $\mathcal{F}$ to $\mathcal{F}$ and from $\mathcal{K}$ to $\mathcal{K}$

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The following result shows that compact dilations model the ***geographical maps***, with their discontinuites (fires that stop at a river, for example)

- Let $\delta(x)$ be the disc of centre x and radius r(x). When

  $\mathbf{x \rightarrow r(x)}$ **is u.s.c.** and $\mathbf{r(x) < r_{max} < \infty}$

  then both δ and ζ are compact.

Scars

- Does the random spread model fit with actual fires data ?

- We can match the « ***scars*** » left by the fires union Y_n of all spreads X_i from steps 1 to n

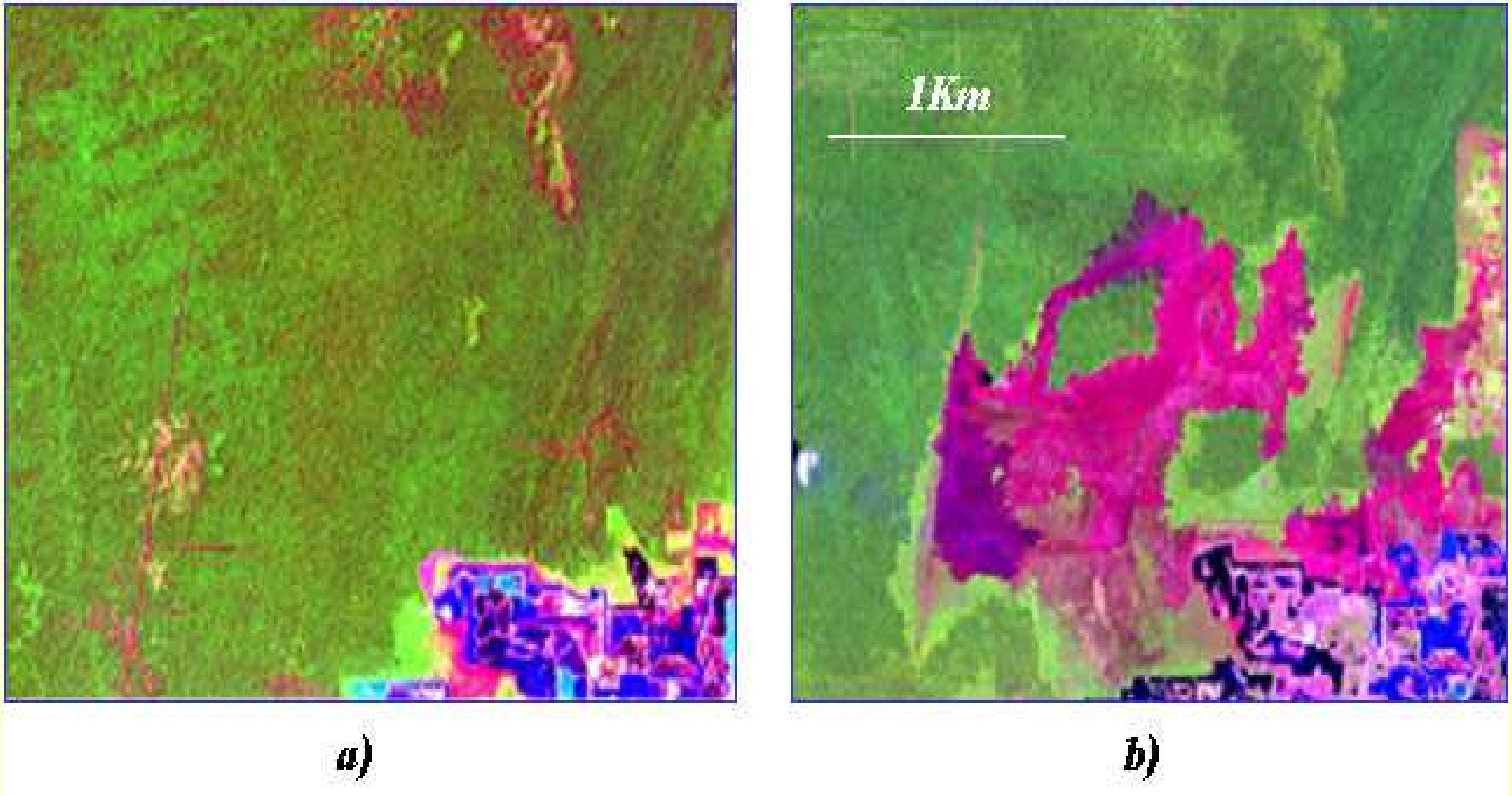
$$\mathbf{Y_n = \cup \{X_n , 1 \leq i \leq n \}}$$

- But what happens after a long time, for $\mathbf{Y\infty}$?

Does the fire stop ? Does it expand indefinitely?

Scars

Example of a scar : A same region in 2000 and in 2004



Upper bounds

For finding an upper bound the scar Y_n , introduce the parameter

$$s(x) = \int_{\delta(x)} \theta(dx)$$

- When $s(x) < s_{max} < 1$ then the scar Y_n is upper bounded by the Boolean RACS of primary grain $\delta(x)$ and of intensity

$$\theta (x) / 1 - s_{max}$$

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- When $s(x) \leq s_{max} < 1$ then the scar Y_n is upper bounded by the Boolean RACS of primary grain $\delta(x)$ and of intensity

$$\theta (x) / 1 - s_{max}$$

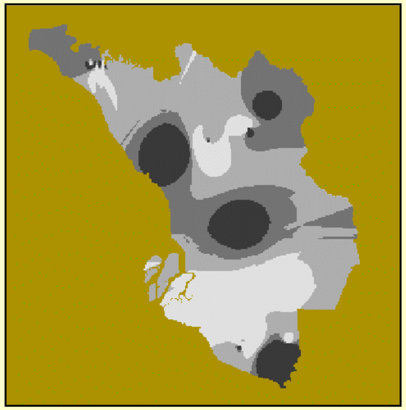
- When not, the scar can expand indefinitely.

This suggests to compare the map of $s(x)$ with the actual scares.

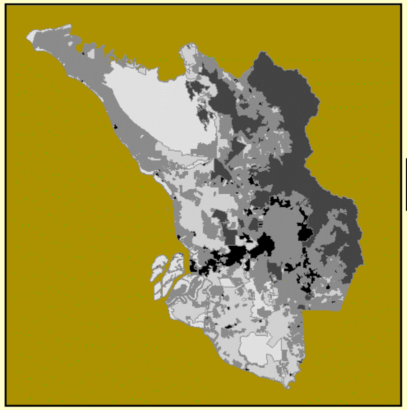
Scar function

The scar function is the product of our two input maps

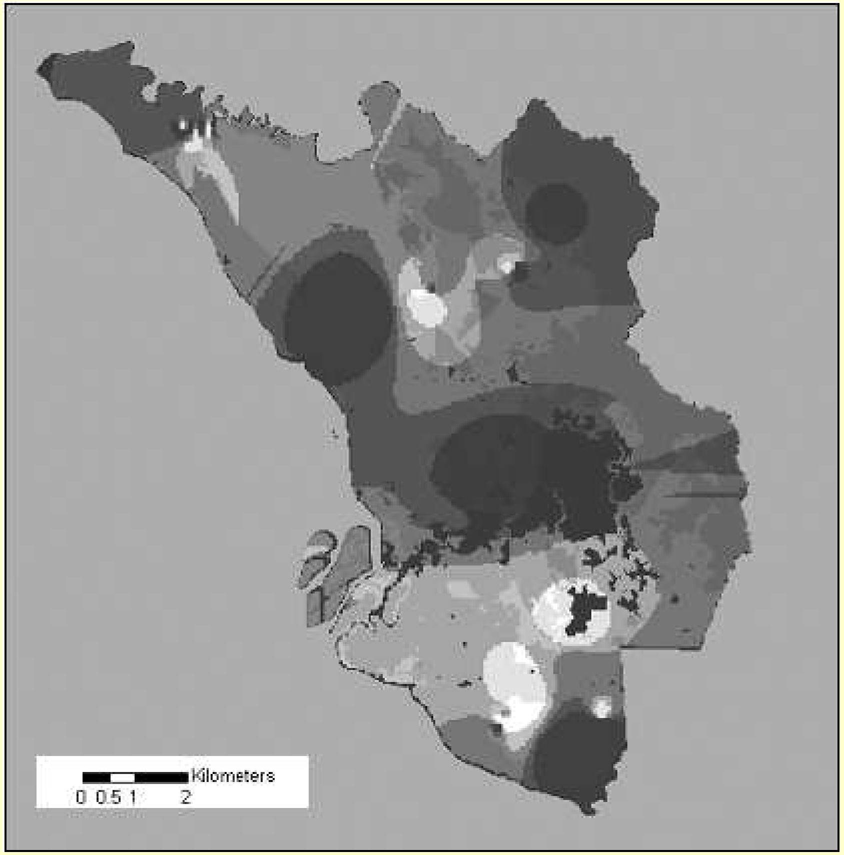
$$s(x) = 2\pi \, . \, \rho(x) \, \theta(x)$$



Spread radius ρ



Fuel amount θ / k



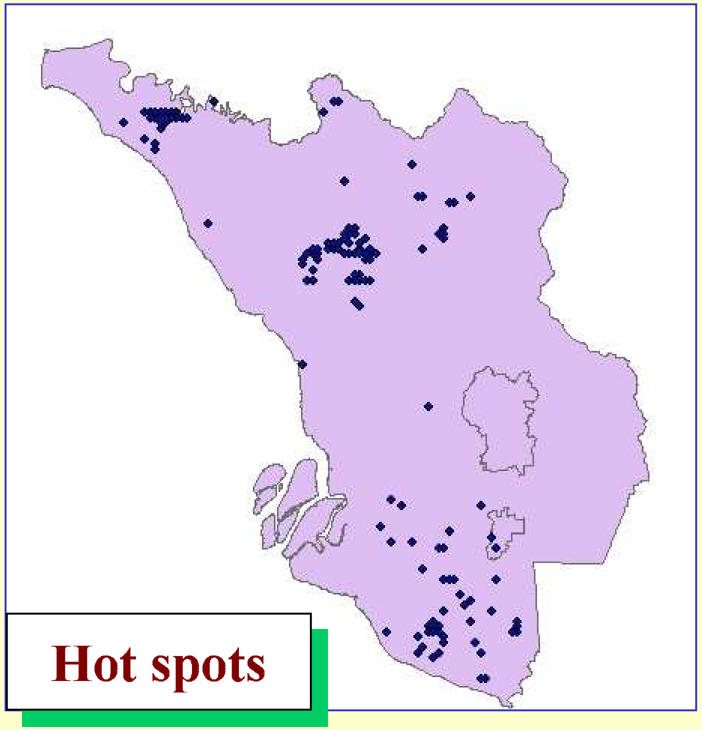
Scar function of Selangor

Hot spots

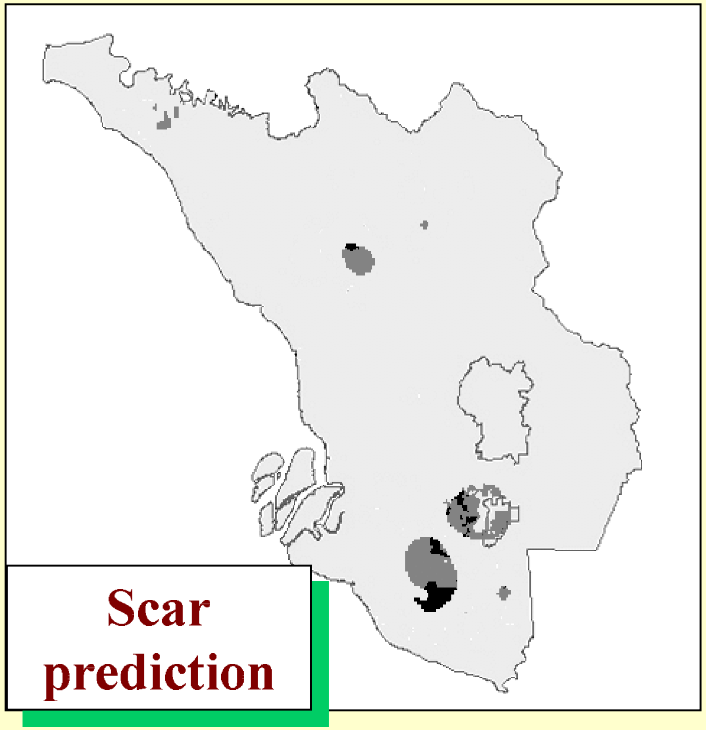
- We obtain a predictor of the scars by ***thresholding***

the scar function **s** above k,

- The seasonal parameter k is estimated by the ***hot spots number***

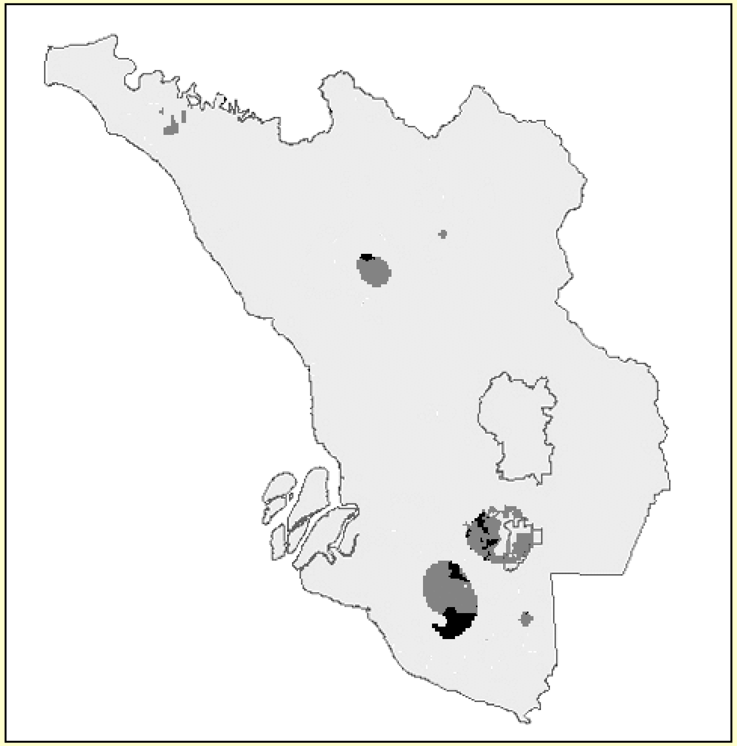


Hot spots

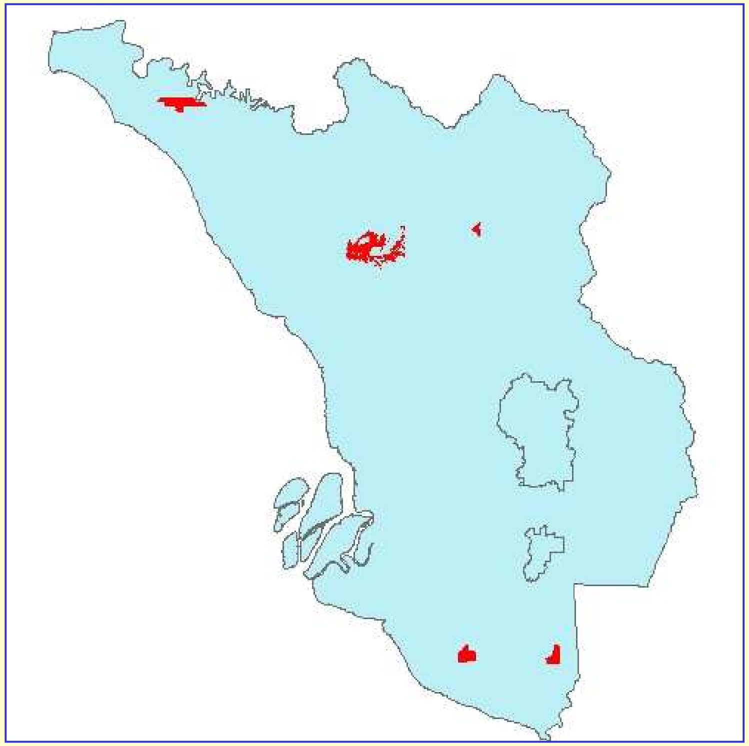


Scar prediction

Results



Scars
From model prediction



Scars
from satellite detection

Period 2001-2004

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- We proposed a new random set which extend the hierarchical structure of some random points to “thick” sets.
- This approach relies on the stochastic model of ***Random Spread*** , which generalizes ***Boolean random set.***
- For forest fires, it results in correct predictions of the ***scars***.
- The model is currently tested on the daily spreads.

Thank you very much

for your attention !

References

- **R. Blanchi, M. Jappiot, D. Alexandrian,** 2002. Forest fire risk assessment and cartograph, a methodological approach. *Forest Fire Research and Wildland Fire Safety*, Viegas Ed. Millpress Rotterdam.
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